Why the fourth prime Hankel determinant vanishes

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Abstract

The same linear relation holds across all four rows. It shows directly that the fourth column is a combination of the first three.

Statement

Number the primes from $p_0 = 2$. For $n \geq 1$, form the $n \times n$ matrix whose entry in row i and column j is p_{i+j}, with both indices ranging from 0 to $n-1$. Let $a(n)$ be its determinant and set $a(0) = 1$. The submitted step is $a(4) = 0$. [1]

Proof

The matrix has rows (2, 3, 5, 7), (3, 5, 7, 11), (5, 7, 11, 13) and (7, 11, 13, 17). Multiply the first entry of each row by 6, the second by -3, the third by -2, and the last by 1. The four sums, in order, are $12 - 9 - 10 + 7$, $18 - 15 - 14 + 11$, $30 - 21 - 22 + 13$, and $42 - 33 - 26 + 17$. Each sum is zero.

Consequently, if C_1, C_2, C_3, C_4 are the columns, then $C_4 = -6C_1 + 3C_2 + 2C_3$. A matrix with a column that is a linear combination of its other columns has determinant zero. This proves $a(4) = 0$. [3]

Context and attribution

Jeffrey Shallit contributed the sequence. Jon Perry conjectured in 2004 that 4 is its only zero. The formal-conjectures library already contained the explicit null-vector calculation as theorem a_4, and the merged submission applies that theorem. An earlier proof submission is linked below. [1, 2]

References

[1] OEIS Foundation. A024356. Sequence definition, conjecture and contributor history.
[2] Open Math Agents. Earlier submission for this reduction step, PR 2176.
[3] Open Math Agents. Merged proof submission 5195 and verification history.
[4] Left to Solve · statement of the submitted problem