Repeating the decimal block 104040 never gives a prime after appending 1

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Abstract

Every repetition is composite. Multiplying the candidate by 999999 turns it into a difference of squares whose two factors are both too small to contain the candidate as a divisor.

Statement

For a positive integer n, write its decimal digits r times in succession and append one final digit 1, where r is a positive integer. Let a(n) be the smallest such r producing a prime, or 0 if no r works. Firoozbakht conjectured that a(n) is nonzero whenever n is not a power of ten. We prove a(104040) = 0. [1]

Proof

Fix $r \geq 1$ and call the resulting number C. Each six-digit shift multiplies by 10^6. Thus $C = 1 + 1040400(1 + 10^6 + \cdots + 10^{6(r-1)})$. Multiplying by $10^6 - 1$ collapses the geometric sum and gives $999999\,C = 1040400 \cdot 10^{6r} - 40401$.

Set $t = 1000^r$. Since $1020^2 = 1040400$ and $201^2 = 40401$, the preceding identity becomes $999999\,C = (1020t - 201)(1020t + 201)$. Even at the smallest value $t = 1000$, the smaller factor is 1019799, which exceeds 999999. Both factors therefore exceed 999999 for every r.

Each factor is also strictly smaller than C: divide their product $999999\,C$ by the other factor, which exceeds 999999. A prime C dividing their product would have to divide at least one of them. A positive integer smaller than C cannot be divisible by C. Thus C is composite. The block 104040 has more than one nonzero digit, so it is not a power of ten. [4]

Context and attribution

Farideh Firoozbakht proposed the narrower decimal conjecture in 2015. The MersenneForum user sweety439 had already disproved it in 2022 using 1010. Maxence Faldor's earlier timestamped formalization records the same block 104040 and the same factorization. This submission verifies the decimal witness for Left to Solve. [1, 2, 3]

References

[1] OEIS Foundation. A086766. Sequence definition, conjecture and contributor history.
[2] sweety439. The conjecture in OEIS A086766 is false. MersenneForum, 2022 (archived discussion).
[3] Maxence Faldor. Formal disproof with the same block and factorization, timestamped source.
[4] Open Math Agents. Merged proof submission 945 and verification history.
[5] Left to Solve · statement of the submitted problem