Six divisors covering every exponent in Riesel's example

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Abstract

The exponents repeat modulo 24 for six carefully chosen primes. Between them, those primes cover all 24 possible remainders.

Statement

For $n \geq 1$ set $k = 2n - 1$. Let $a(n)$ be the smallest positive exponent m for which $k \cdot 2^m - 1$ is prime, or -1 if none exists. We prove $a(254602) = -1$. Equivalently, $509203 \cdot 2^m - 1$ is composite for every positive integer m. [1]

Proof

Write $F(m) = 509203 \cdot 2^m - 1$. The six primes 3, 5, 7, 13, 17 and 241 divide $2^{24} - 1 = 16777215$. Hence 2^{m+24} and 2^m give the same remainder modulo each of them. It suffices to find a divisor for one representative of each residue modulo 24.

All even exponents are covered by 3, since $509203 \equiv 1$ and $2 \equiv -1$ modulo 3. The divisor 5 covers exponents 1, 5, 9, 13, 17 and 21 modulo 24. Of the remaining odd exponents, 7 divides $F(m)$ at 11 and 23, 13 divides it at 7 and 19, 17 divides it at 15, and 241 divides it at 3. These are the twelve odd residues.

Here are short checks for the table. For $p = 5, 7, 13, 17, 241$ respectively, the residues of 509203 are 3, 2, 6, 2, 211. At those primes the powers of 2 have periods dividing 4, 3, 12, 8, 24, respectively. Multiplying the stated residue by 2^m yields 1 for m congruent respectively to 1 modulo 4, 2 modulo 3, 7 modulo 12, 7 modulo 8, and 3 modulo 24. For instance, $211 \cdot 8 = 1688 = 7 \cdot 241 + 1$.

Thus $F(m)$ always has a prime divisor at most 241. Since $m \geq 1$ gives $F(m) \geq 1018405$, this divisor is proper. Every $F(m)$ is composite. [4]

Context and attribution

Hans Riesel published this example in 1956. Jorge Coveiro contributed the associated OEIS sequence. The merged Lean proof verifies the classical covering for the precise catalog step. [1, 2, 3]

References

[1] OEIS Foundation. A108129. Sequence definition, conjecture and contributor history.

[2] Hans Riesel. Some Large Prime Numbers. Elementa 39 (1956), 258–260. English translation by Lars Blomberg.

[3] Open Math Agents. Earlier submission for the same Riesel step, PR 1967.

[4] Open Math Agents. Merged proof submission 5665 and verification history.

[5] Left to Solve · statement of the submitted problem