Five rows of Pascal’s triangle with odd binary digit sums

Anmol Sahu · 28 November 2026

Abstract

Row symmetry reduces the five checks to nine different positive integers. Each has either one or three ones in binary.

Statement

Call a nonnegative integer odious if its binary expansion contains an odd number of ones, and evil if that count is even. For row n of Pascal’s triangle, let a(n) be the first index m with $1 \le m \le n$ for which the binomial coefficient C(n,m) is evil, or 0 when no such index exists. We prove $a(n) = 0$ for n in {0, 1, 2, 7, 8}. [1]

Proof

Number the rows starting with row 0 = (1). Because $C(n,m) = C(n,n-m)$, it is enough to inspect each row from its left edge through its middle. For rows 0, 1 and 2 the lists are (1), (1) and (1, 2). For row 7 the list is (1, 7, 21, 35). For row 8 it is (1, 8, 28, 56, 70).

The values 1, 2 and 8 are powers of two, so each has exactly one binary one. The other six values are $7 = 111$, $21 = 10101$, $35 = 100011$, $28 = 11100$, $56 = 111000$ and $70 = 1000110$, where the digits on the right are in base two. Each contains three ones. Every entry in the five rows is therefore odious. At row 0 the defining search for a positive index is empty; it too gives $a(0) = 0$. [3]

Context and attribution

Vladimir Shevelev proposed the classification in OEIS A249609. The catalog also records larger searches by Peter J. C. Moses and Michael S. Branicky. The merged proof checks only the five stated rows. An earlier submission addressed the same step. [1, 2]

References

[1] OEIS Foundation. A249609. Sequence definition, conjecture and contributor history.
[2] Open Math Agents. Earlier submission for the five specified rows, PR 2211.
[3] Open Math Agents. Merged proof submission 5281 and verification history.
[4] Left to Solve · statement of the submitted problem