The real part of a Möbius sequence

Anmol Sahu · 28 November 2026

Abstract

The real part of i raised to n depends only on the remainder of n modulo four. In each of the four cases its product with $\mu(n)$ gives the defining value of the sequence.

Statement

For $n \geq 1$ let $\mu(n)$ be zero if a prime square divides n, and otherwise let $\mu(n) = (-1)$ raised to the number of distinct prime factors of n. Set $a(n) = 0$ when n is odd and $a(n) = -\mu(n)$ when n is even. Then $a(n)$ is the real part of $\mu(n)$ times i raised to n, where $i^2 = -1$. [1]

Proof

Starting from $i^0 = 1$, successive powers cycle through 1, i, -1 and $-i$. Because $\mu(n)$ is a real integer, the real part of its product with a power of i is $\mu(n)$ times the real part of that power.

When n is odd, the power is i or $-i$, both with real part zero. For n congruent to 2 modulo 4 the power is -1, giving $-\mu(n)$, which is the even-index rule.

Finally, when 4 divides n, the power is 1. The square of the prime 2 divides n, so $\mu(n) = 0$. Both the product and the even-index rule then give zero. These cases exhaust all $n \geq 1$. [5]

Context and attribution

Antti Karttunen contributed the sequence to OEIS. Mats Granvik proposed the real-part identity in 2024. The project credits the proof submitted under anmolspace with Claude Code. An earlier submission by xor2k addresses the same theorem. [1, 2, 3]

References

[1] Mats Granvik, OEIS A292273, real-part formula
[2] OEIS A008683, Möbius function
[3] xor2k, earlier project submission for A292273, PR 7021
[4] Left to Solve · statement of the submitted problem
[5] Open Math Agents · merged proof, pull request #8351