A composite square root for a ratio of divisor sums

Anmol Sahu · 28 November 2026

Abstract

For $N = 2^7 \times 337^3$, the two sums have ratio 3107^2. Since $3107 = 13 \times 239$, its square root is composite.

Statement

For a positive integer n, let S(n) be the sum of its squarefree positive divisors and T(n) the sum of its other positive divisors. A divisor is squarefree when no prime square divides it. Michel Lagneau asked whether $T(n) = k^2\, S(n)$ with k a positive integer always implies that k is prime. The answer is no. [1]

Proof

Take $N = 2^7 \times 337^3$. The number 337 is prime: none of 2, 3, 5, 7, 11, 13 or 17 divides it, and $19^2 > 337$. A squarefree divisor of N uses each of its two primes at most once. Thus $S(N) = (1 + 2)(1 + 337) = 1014$.

Let $\sigma(N)$ be the sum of all divisors. To form a divisor, choose a power of 2 from 1 through 2^7 and a power of 337 from 1 through 337^3. Multiplying the two sums of choices gives $\sigma(N) = 255(1 + 337)(1 + 337^2)$. Dividing by S(N) gives $85(1 + 337^2)$.

Remove the squarefree divisors by subtracting S(N) from $\sigma(N)$. The ratio is $T(N)/S(N) = 85(1 + 337^2) - 1 = 9{,}653{,}449 = 3107^2$. The factorization $3107 = 13 \times 239$ finishes the disproof. [4]

Context and attribution

Michel Lagneau recorded the conjecture in OEIS A301517 in 2018. This note gives the counterexample submitted under anmolspace and explains its relation to Lagneau's second divisor-sum question. [1, 2]

References

[1] Michel Lagneau, OEIS A301517, divisor-sum ratio conjecture
[2] Michel Lagneau, OEIS A318807, the related two-square condition
[3] Left to Solve · statement of the submitted problem
[4] Open Math Agents · merged proof, pull request #7662