# Primitive digit blocks with composite repetitions in every base

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**Abstract**

Repeat a positive integer's digit block any positive number of times, then append the digit 1. In every base $b \geq 2$, we construct infinitely many blocks for which the resulting integer is always composite. Each block begins with 1, contains another nonzero digit, and is primitive: it cannot be written as copies of a shorter block. The construction comes from a difference of squares and gives an explicit family of exceptions to a proposed refinement of the conjecture associated with OEIS A086766.

## The construction

For an integer $n > 0$ written in base $b$ without leading zeros, let $C_{b,r}(n)$ be the integer obtained by writing its block $r$ times and appending 1 in the same base. For instance, $C_{10,2}(12) = 12121$. Firoozbakht asked whether every decimal input other than a power of ten gives a prime for some $r$ [1]. In 2022, the MersenneForum user sweety439 gave the counterexample 1010 and discussed the possibility of excluding repeated copies of a 1 followed by zeros [2]. The following family survives that exclusion in every base.

**Theorem.** *Fix integers $b \geq 2$ and $k \geq 1$ with $x = b^k \geq 3$, and set*

$$n = \frac{(x^3 + 2x)^2}{b}.$$

*The base-$b$ expansion of $n$ is a primitive block of length $6k$. Its leading digit is 1 and it has another nonzero digit. For every $r \geq 1$, $C_{b,r}(n)$ is composite.*

**Length and leading digit.** Since $b \mid x$, the displayed value of $n$ is an integer. Its ratio to $b^{6k-1}$ is $1 + 4/x^2 + 4/x^4$, strictly between 1 and 2 for $x \geq 3$. Thus $n$ has exactly $6k$ base-$b$ digits, the first is 1, and the other digits cannot all vanish.

**Compositeness.** Put $A = x^3 + 2x$, $D = 2x^2 + 1$, and $M = x^6 - 1$. Direct expansion gives $M = A^2 - D^2$. Consecutive copies of the block shift by a factor of $x^6$, so the geometric sum and $bn = A^2$ give

$$MC_{b,r}(n) = bn(x^{6r} - 1) + M = (Ax^{3r} - D)(Ax^{3r} + D).$$

Both factors exceed $M$: even at $r = 1$ the smaller factor exceeds $M$ by $2x^2(x^2 - 1)$. Consequently each factor is smaller than $C_{b,r}(n)$, since the other factor exceeds $M$. If $C_{b,r}(n)$ were prime, it would divide one of these two smaller positive integers. This proves compositeness.

**Primitivity.** Suppose the block consists of $s \geq 2$ copies of a word of length $d$, so $sd = 6k$. The last $2k-1$ digits of $n$ are zero because $b^{2k-1} \mid n$. If $s \geq 4$, then $d \leq 3k/2 < 2k$. The final word would consist entirely of zeros, contradicting the leading digit of its first copy.

If $s = 2$, the two copies imply $x^3 + 1 \mid n$. Modulo $x^3 + 1$ we have

$$bn = (x^3 + 2x)^2 \equiv (2x - 1)^2.$$

The residue is positive and smaller than $x^3 + 1$, since $x^3 + 1 - (2x-1)^2 = x(x-2)^2 > 0$. This rules out $s = 2$.

If $s = 3$, the three copies imply $x^4 + x^2 + 1 \mid n$. Modulo this divisor we have $x^6 \equiv 1$ and $x^4 \equiv -x^2 - 1$. Hence

$$bn = x^6 + 4x^4 + 4x^2 \equiv -3.$$

The divisor exceeds 3, giving the last contradiction. For every fixed base there are infinitely many eligible $k$, and their different block lengths give different examples. □

**Examples and prior work.** With $b = 2,\ k = 2$, the construction gives $n = 2592$, whose binary block is 101000100000. With $b = 10,\ k = 1$, it gives 104040 and

$$999999\, C_{10,r}(104040) = (1020 \cdot 1000^r - 201)(1020 \cdot 1000^r + 201).$$

Faldor's timestamped Lean development records this same decimal block and factorization [3]; it predates our public submission of the decimal case [4]. The 2022 discussion also gives factorizations in several other bases [2]. The theorem above gives one construction with primitivity and infinitude simultaneously in every base.

**Proof files and contributions.** The companion file `RepeatedBlockFamily.lean` verifies the digit length, primitivity of the canonical digit list, and compositeness of the repeated-block construction in Lean 4. Anmol Sahu directed the project. OpenAI Codex assisted with the mathematical argument and its formalization and drafted this standalone exposition. The cited public records identify the earlier examples.

## References


[1] OEIS Foundation, A086766, conjecture of Farideh Firoozbakht (2015).

[2] sweety439, The conjecture in OEIS A086766 is false, MersenneForum (2022).

[3] M. Faldor, Lean disproof of the A086766 conjecture, public build, 26 November 2026.

[4] A. Sahu, Disproof with the decimal block 104040, Open Math Agents, PR 945, 27 November 2026.